Mathematical Methods in Economics
Syllabus

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Version: August 4, 2020

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Lectures. Mon-Fri: 10:00–12:00, 13:00–15:00 CDT (may be subject to change)
Zoom link (subject to change): `https://zoom.us/j/ID-NUMBER`

TA sessions. Mon-Fri: 15:30–16:30 CDT (may be subject to change)
Zoom link (subject to change): `https://zoom.us/j/ID-NUMBER`

1 Introduction

The primary objective of this course is to introduce and/or review mathematical concepts that will be useful for the first-year sequence for the PhD in Economics. Reflective of the types of questions that you will be solving over the coming year, we will take a *practical approach*, focusing on the statement of useful results and their applications, rather than their proofs. We suspect many of you have would have had varying degrees of exposure to mathematics at different points in your life and that you may well have forgotten some concepts. Therefore, we will cover some "basic" mathematics (albeit briefly) as a refresher. As such, there is no need for you to prepare for this class in advance.

2 Lectures and TA sessions

As we find ourselves in a continuing global pandemic, the course will be taught via Zoom and the relevant materials will be made available via Canvas.

Classes will be held every weekday beginning on 31 August and run until 18 September (except 7 September, Labor Day). Each weekday, there will be two lectures from 10:00–12:00 and 13:00–15:00 (CDT). The lectures may be followed by a TA session from 15:30–16:30 (CDT); we will make further announcement as to the schedule of TA in due course.

If you have any questions regarding the materials, we ask you to post them in the dedicated Slack group (link subject to change) – we hope that this encourages the spirit of community learning and public service (as it is rare that questions aren't shared by many others). Please contact the lecturers or the TA directly in case you have administrative questions

3 Grading and problem sets

There is no grade for this class and the validation is pass/fail (with "passing" being almost sure).

Nevertheless, there will be two or three *problem sets* with theoretical, empirical and numerical exercises, aimed at putting the mathematical concepts that we cover in lectures in practice. The problem sets are intended to give you a chance to warm up and learn/practice some skills you will use during the next few years.

The problem sets are not compulsory and will not be graded although we will provide solutions. However, fully grasping mathematical concepts requires learning-by-doing and solving the problem sets will help you to this end (even if you have covered the topics in the past). If you are not very familiar with the materials covered in class, these few weeks will be the only weeks that you can have chances to review these tools in details, before the madness of the academic year begins. We encourage you to work in groups which should facilitate formation of study groups, which will become important during your first year.

4 Syllabus

The course is split into four parts: A. foundation; B. static optimisation; C. probability theory; and D. control theory and model simulations. Takuma Habu will cover parts A, B and the first half of C, and the rest will be covered by Thomas Bourany. A tentative schedule of topics that will be covered is given below.

Part A: Foundation

(i) Real analysis Fundamentals (logic, proofs, sets, relations, functions, spaces); Metric spaces (sequence and function spaces); Open/closed sets; Sequences and convergence; Boundedness and compactness; Completeness; Contraction Mapping Theorem; Continuity and Continuous functions; Intermediate Value Theorem and Weirstrass Theorem.

(ii) Linear algebra Vector spaces; Linear dependence; Inner products/norms; Matrices, span, basis, inverse; System of linear equations; Eigenvalues, eigenvectors, diagonalisation and spectrum theorem; Definiteness/quadratic forms; Projections.

(iii) Correspondences* Hemicontinuity; Brouwer/Kakutani's fixed point theorems; Berge's maximum theorem.

Part B: Static optimisation

(i) Calculus Derivatives and Jacobian; Mean Value Theorem, Taylor's expansion (O, o); Implicit function theorem; Inverse function theorem; (Riemann-Stieltjes) Integration (by parts); Homogeneity and homotheticity; L'Hopital rule; Leibniz rule; Fundamental Theorem of Calculus; Young's Theorem; Vector/Matrix calculus;.

(ii) Convexity Convex sets; Separation Hyperplane Theorems; (Quasi-) convex/concave functions.

(iii) Static optimisation Unconstrained; KKT; Envelope Theorem; Theorem of Maximum; Uniqueness and sufficiency of FOC.

Part C: Probability theory

(i) Basic Probability Theory (Recap) Probability, Bayes' rule; Random variable, CDF/PDF; Expectation, Variance, Covariance, Correlation; CDF/PDF; Independence; Conditional expectations; Distributions (normal, Poisson, exponential, binomial); Inequalities (Triangle, Cauchy-Schwarz, Jensen, Chebycehv, Markov); Stochastic order (FOSD, SOSD); O_p, o_p and operations; LLN and CLT

(ii) Measure theoretic probability Measure/probability spaces; Measurability; Lebesgue integration; Absolute continuity; Fundamental Theorem of Calculus; Convergence theorems; Conditional expectations; Types of convergence (probability, L^p, distribution, almost sure).

(iii) Numerical probabilities and applications Random variables simulations and convergence; Monte-Carlo based methods; Conditional expectation; Gaussian vectors and conditional law; Estimation: GMM methods.

(iv) Stochastic processes and stochastic calculus Markov chains; Martingales; Continuous-time stochastic processes; Stationary distribution and Kolmogorov Forward equations.

Part D: Control theory and model simulations

(i) Control: Sequential formulation Pontryagin maximum principle; Discrete time and Euler equations; Continuous time and ODEs.

(ii) Control: Recursive formulation Dynamic programming à la Bellman; Discrete time; Continuous time: HJB and primer on PDEs.

(iii) Numerical methods for model simulation* Gradient descent and Newton methods; Shooting algorithm; Dynamic programming/Value function iteration; Finite difference methods; Perturbation methods.